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SOLVING DYNAMIC MODEL OF INFORMATION DISSEMINATION IN CAPITAL MARKETS

Abstract. *The main reason why companies decide to realize with IPO is to gain access to new funding and they want to signal its quality to the market. The publication presents a model of information dissemination in capital markets. We look upon dissemination of the information as similar to viral infection in epidemiology.*

The aim of the paper is to use the newly created mathematical apparatus enabling the analysis of dynamic models' behavior, and more complex and detailed study of their behaviors even if some of the variables respond to stimuli with non-constant time delay. This procedure involves the use of the a priori solution estimate method to design an original solution of boundary value problems for equations (systems) with delay(s) through a sequence of simpler boundary value problems without delay. The results are demonstrated on a specific example and the behavior of the model is presented using a computer simulation.

Keywords*: capital market; Initial Public Offering; nonlinear ordinary differential equations.*

JEL Classification : C02, C11, C45, C46, C63

1. Introduction

The world is perceived as increasingly complex and so are the means we use when trying to describe and control the surrounding world. With the advent of computers, new possibilities have been opened enabling both more efficient use of classical tools (for example, solutions to large systems of equations) and entirely new approaches, such as modelling and simulation. Modelling and simulation

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provide us with an effective tool to understand and deal with the behavior of more complex systems.

In the field of social systems, especially in economics, modelling has long been used. Classical economic models have been inspired by physical models of simple systems and consist of a few equations for which we analytically seek solutions (typically equilibrium). However, in many situations, classical models are inappropriate, and we need to look at economy as a complex system.

The main aim of the paper is to describe a newly designed procedure that allows us to analyze and solve dynamic models, by using numerical mathematical tools, in which the time delay of variables must be considered. Part of the paper describes construction of this solution of the nonlinear dynamic model using the method of successive approximations, which is based on the modern theory of functional differential equations. Verifying the solvability of this problem by means of a new procedure is the scientific objective of the paper.

The new procedure will be demonstrated on the solution of the model of dissemination of information on the launch of a new stock on the stock market, especially while the new stock is presented to potential investors (so-called Road Show). Our model is inspired by viral dynamics and is described by a nonlinear system of differential equations with a delayed argument.

The behavior of the model in case of different input values is simulated in the Maple system and graphically presented within the application section.

2. Literary Review

Currently, some authors are demonstrating that it is especially the category of differential equations with delayed argument that is suitable to describe the dynamics in which past states affect the current state of a system (e.g. Matsumoto and Szidarovszky, 2013; Dzhalladova et al., 2019; Lutoshkin and Yamaltdinova, 2018).

Available literature on the solvability of differential equation systems with delayed arguments offers a number of results useful in practice. An extensive and comprehensive analysis of such equations can be found notably in (Bellen and Zenaro, 2013; Ravi and Murali, 2018) and (Kiguradze and Půža, 2003) and the literature cited therein. The conditions of solvability, i.e. existence and uniqueness, of both general and special problems, conditions of their correctness (i.e. little dependence of a solution on "small" changes in initial conditions and parameters necessary for numerical solution), and conditions of non-negativity of a solution and others are studied in detail.

However, insufficient attention is paid to the systematic description of methods of constructing a solution of problems with delays and to the analysis of their usability in as much detail as in "standard" ordinary differential equations, except for work by (Bellen and Zenaro, 2013) which uses Runge-Kuta modifications of "local" methods of constructing solutions of equations with delay, (Gelashvili and Kiguradze, 1995) which contains derivation of differential schemes

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of global approach applications in the Carathéodory theory and (Půža and Novotná, 2018) which deals with the construction of a solution of a linear dynamic model using the method of successive approximation. The theory presented in publications (Kiguradze, 1988) and (Kiguradze, 1997) is based on the methods of a priori estimates of solution of functional differential equations and their systems, and it forms a suitable basis for using the method of successive approximations in an appropriate associated operator equation with a contractive operator.

Nowadays, however, a wide-ranging systematic apparatus is missing to perform numerical calculations for a nonlinear dynamic model described by a system of ordinary differential equations with delayed (non-constant) arguments, which, at the same time, would not depend solely on specialized software applications and could be used wherever any program is available for a numerical solution of an initial problem for ordinary differential equations and their systems.

3. Framing a problem of the model

Many theories about the IPOs' assume the existence of asymmetric information because the parties involved in the process of introducing new shares of the company in the stock exchange, do not hold the same amount of information. Stock exchanges in advanced economies have gradually changed from present stock exchanges to the electronic stock exchanges. Securities market participants are well informed (we assume a light form of the weak form efficiency hypothesis) and use increasingly more advanced types of models to manage investment and financial risks, investment analysis and asset management. As follows from papers (Escobari and Jafarinejad, 2019) and (Wu et al., 2020), it is necessary, in stock market conditions, to focus primarily on non-linear dynamic models.

Timing and the effect of time delay in the stock market were studied in (Miswanto, 2018) and (Wu et al., 2020), which confirm its significant influence on the model's behaviour. The actual process of launching a new share issue on the stock exchange and the public market were dealt with in (Meluzín et al., 2018) and (Škapa and Meluzín, 2011).

In the Road Show process (i.e. presentation of a company to potential investors), it is crucial to raise interest among potential investors. The way people are influenced by new information can be described, for example, by using the 2D ideal gas model (Bao and Frichtman, 2018) or as a process analogous to the process of spreading viral infection in the theory of viral dynamics. A very similar example is the spread of some type of behavior (e.g. spreading a game strategy, political opinion, new technology or fashion product, etc.). These findings, among others, led to the development of the theory of viral marketing and viral advertising (Sharma and Kaur, 2018; Sela et al., 2018), which triggered revolutionary impacts on marketing areas. Viral marketing is a method used to achieve exponential growth of brand awareness (or a product or service) through the unmanaged dissemination of information among people. It spreads like wildfire and can be likened to a viral epidemic - hence the name of this method (Hao and Wang, 2016).

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In his paper (Walter, 2019), Walter verifies the hypothesis that the principle of viral dynamics in the financial sector is more suitable for modelling than the principle of Brown's motion.

During the IPO, there is high information asymmetry among investors and issuers and from the investors' point of view, IPOs are particularly interesting due to two anomalies associated with them. From the investors' point of view, IPOs are particularly interesting due to two anomalies associated with them. Most IPOs are underestimated, which means investors can make profits on the first day. Another interesting point is that companies going public are less efficient in (about) the following five years than the average established by the market index (Meluzín et al., 2018).

2.2. Model of information dissemination in capital markets

Models of a network of virus populations and immune responses which consist of systems of differential equations can be found, for example, in (Browne and Smith, 2018; Sharma and Kaul, 2018). The basic idea of the model is that a group of infected individuals is considered in a population (people, animals, etc.). What we are interested in is the spread of the infection in this population as a function of time. To take the model of viral dynamics, found for example in (Nowak and May, 2000), but we will consider the fact that the model does not include incubation time.

Let us assume on the sequel process can be seen as analogous to the viral marketing process. We now divide it into several phases, more detailed analysis of which can be found in (Hao and Wang, 2016). Factors such as sensitivity to information, investor behavior, investors' views and mental state, and the atmosphere of the stock market impact on the stock market significantly, so the "infection process" may be considered rather complicated.

The individual phases may be briefly described as "Awareness" – potential investors have received information on the issue of new shares and process it in their subconscious, "Evaluation" – potential investors consider their options and assess if investment might be profitable for them, "Investment" – in the stage a potential investor has already made a decision, makes the investment and becomes a shareholder.

The model has variables – "the number of potential investors", which we denote by $I(t)$, and "the number of shareholders" denoted by I^*. The model also assumes that each investor only invests in one share.

Let us now start from the premise that the initial issue of shares was offered to several institutional investors during the Road Show and that trading commenced on the stock exchange at time $t = 0$ and that is when small investors, or physical persons, could start purchasing the shares. Therefore, the intensity of the "signal" S is derived from the amount invested in advertising.

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If we have information about the Road Show, we are able to determine the values of so called historical functions I_h, I_h^* and S_h at time $t = 0$. During the Road Show, potential investors have enough time to decide whether to buy the share, but when the share is already being traded, they need to respond to new information flexibly. Reflection time τ then refers to the time that investors on average need to make decisions at time $t > 0$. The level of potential investors p includes both institutional and private potential investors. Parameter q denotes the number of investors leaving the market.

To bring the model closer to reality, we will also include an increase in the number of potential investors $\xi I(t)\left(1-\frac{I(t)}{I_{max}}\right)$, where ξ is the growth rate of potential investors, i.e. it determines how the number of people considering investment in the shares listed is increasing. Typically, the parameter is higher than q. I_{max} is the maximum stock market capacity, expressing the number of shares available to shareholders, i.e. shares that are not owned by a strategic owner.

The model should also take into account the market sensitivity to information, which is expressed by positive constant β. The constant determines how many potential investors become shareholders in relation to the strength of signal S.

$\frac{dI(t)}{dt}$ represents the dynamics of potential investors.

As mentioned above, there is a time delay between receiving information and making a decision, caused by the fact that a potential investor needs time to evaluate the situation. (Meluzín a kol., 2018).

For this reason, it is necessary to consider the effect of time delay in the model. If a potential investor has captured a signal at time $t-\tau(t)$, the signal will be evaluated on the interval $(t-\tau(t), t)$. Of course, the decision may be either negative – not to invest, or positive – to invest free funds in the share. The increase in the number of shareholders given by the signal of a new share issue, is denoted $\beta I\big(t-\tau(t)\big)S\big(t-\tau(t)\big)$. Another option is that the investors sell the shares they own, having considered all the circumstances. We call the rate of natural decrease in the number of shareholders $\delta(t)$. The term $\frac{dI^*(t)}{dt}$ represents the dynamics of stockholders.

Natural decrease in the number of shareholders will now be related to the rate of forget fulness and will be considered a function of time. Let us assume that at time $t = 0$ the value of the information considered is equal to δ_0 and its decrease is proportional to the actual size of information $\delta(t)$ at time t.

$\frac{d\delta(t)}{dt} = -n\delta(t)$, where $n > 0$ is a suitable constant. This assumption was supported by experiments which confirmed that information is not forgotten linearly over time.

Dependence correlates with a so-called “pure forgetting” curve. This topic was studied, for example, in (Šimon and Bulko, 2014) . It has always been

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necessary to anticipate nonzero "residual" information in conscious memory. Assuming that loss of information is compensated by receiving new information on the basis of a homeostatic principle, we modify the original assumption to the following form

$$\frac{d\delta(t)}{dt} = c - n\delta(t), \quad c \geq 0,\ n > 0, \tag{1}$$

where *c* represents the speed at which new information is received. We expect both constants to depend on the type of information and the individual. The solution to differential equation (1) is time dependence:

$$\delta(t) = \frac{c}{n}(1 - e^{-nt}) + \delta_0 e^{-nt}, \quad t > 0$$

where the first addend represents a time-dependent increase in information, under given assumption it is $1 - e^{-nt} > 0$, and the second one is "pure forgetting".

Let us mark the rate of signal change as d_3, related both to the amount of advertising investment and the dissemination of information, and to the fact that potential investors might be somewhat resilient to the signal.

It should also be taken into account that information also spreads through interpersonal contact, which, of course, also exists among shareholders and potential investors. (Hoffmann and Broekhuzien, 2009) The degree of contact among shareholders and prospective investors is represented by parameter *k*, i.e. $\frac{dS(t)}{dt}$ represents the dynamics of the signal.

The whole system may be expressed in the following way:

$$\begin{aligned} \frac{dI(t)}{dt} &= p - qI(t) + \xi I(t)\left(1 - \frac{I(t)}{I_{max}}\right) - \beta I(t)S(t) \\ \frac{dI^*(t)}{dt} &= \beta I(t-\tau)S(t-\tau)\delta(t)I^*(t) \\ \frac{dS(t)}{dt} &= -d_3 S(t) + kI(t) \end{aligned} \tag{2}$$

where $p, q, \xi,\ I_{max}, d_3, \beta$ and k are positive constants. System (2) is studied under the condition

$$I(0) = I_0 > 0, I(0) = I_0 > 0, I^*(0) = I_0^* > 0, S(0) = S_0 > 0 \tag{3}$$

where I_0, I_0^* and S_0 are positive constants.

For $t < 0$ it is further assumed $I(t) = I_h(t), I^*(t) = I_h^*(t), S(0) = S_h(t)$, where the historical functions $I_h\ (t)$, I_h^* and $S_h(t)$ are continuous and nonnegative.

3. Construction of a numerical solution

From the mathematical point of view, system (2) describing the dynamics of signal propagation of a new share on the market can be referred to as a nonlinear system of ordinary first-order differential equations with a non-constant delay.

The functions in system (2) are generally unknown functions whose parameters $p, q, \xi, I_{max}, d_3, \beta$ and k are positive constants, $\delta(t)$ for $t > 0$ is a

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continuous bounded function of forgetting, and τ is a delay in the impact of I^* and S on the solution of system (2), In general, this delay may have a major impact on the stability of the system, its periodic behavior, bifurcation, and other dynamic properties.

Furthermore, it may be stated that the system of three equations (2) considered on the interval J= $[0; t_r]$ for a sufficiently large $t_r > 0$ involves a continuous retarded argument and belongs among systems of ordinary differential equations with deviating arguments or, more generally, among the so-called is a special case of a system of functional differential equations

$$x'(t) = f\Big(t, x(t), x\big(\tau(t)\big)\Big), \qquad t \in J, \tag{4}$$

with a measurable argument deviation $\tau: J \to R$ and the function $f: J \times R^{2n} \to R^n$ satisfying the Carathéodory conditions (Gelashvili, 1995; Kiguradze, 1997).

In the framework of the Carathéodory theory, a solution of system (2) is a understood as a vector function $x: J \to R^n$ absolutely continuous on I satisfying (3) for a.e.t $\in J$. Our particular form of system (3) ensure that solutions of (4) is also continuously differentiable on I (therefore, a solution is continuous on this interval together with its derivative, i.e., its graph is smooth).

Let us focus on solutions of system (4) satisfying the condition

$$x(t_0) = c_0, \tag{5}$$

where $t_0 \in J$ and $c_0 \in R^n$ (i.e., consider the initial value problem (3), (5). Due to the character of the problem, the setup should be complemented by adding an initial function describing the solution for times preceding the interval I, i.e.

$$x(t) = h(t) \qquad \text{for } t < 0 \tag{6}$$

where $h: (-\infty, 0) \to R^n$ is continuous and bounded. If a continuous junction of the initial function with the solution at 0, we continuously extend h to $(-\infty, 0]$ and study the initial value problem (3), (5), (6) in the form

$$\begin{gathered} x'(t) = f\Big(t, x(t), \chi_I\big(\tau(t)\big)x\big(\tau^0(t)\big) + \Big(1 - \chi_I\big(\tau(t)\big)\Big)h\big(\tau(t)\big)\Big), \\ x(0) = h(0), \end{gathered} \tag{7}$$

where

$$\chi_I(t) = \begin{cases} 1\, for\ t \in J \\ 0\, for\ t \notin J \end{cases},$$

and

$$\chi_I(t) = \begin{cases} \tau(t)\, for\ \tau(t) \in J \\ 0 \quad for\ \tau(t) < 0 \end{cases}.$$

Problems of type (7) and its extensions are the object of much attention in the literature during the last two decades. Numerous conditions are known which ensure the existence of a solution and its uniqueness, positivity of the solution as well as well-posedness of the various types of boundary value problems for functional differential equations which cover, as a special case, equations with

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retarded argument. However, there is still not sufficiently many effective methods to construct approximate solutions for more or less general classes of problems.

For problem (7), the solution can be obtained by using the following method of successive approximations involving a sequence of solutions of ordinary differential equations without argument deviation.

Theorem

Assume that $\tau\colon \mathrm{J} \to R$ be continuous and $\tau(\mathrm{t}) \leq \mathrm{t}$ for $t \in J$, the function $\mathrm{h}\colon(-\infty, 0] \to \backslash R^n$ is continuous and bounded, the function $\mathrm{f}\colon \mathrm{I} \times R^{2n} \to R^n$ is (piecewise) continuous with respect to the first variable, continuous with respect to the other *2n* variables, and $|\mathrm{f}(\mathrm{t},\mathrm{x},\mathrm{y}) - \mathrm{f}(\mathrm{t},\bar{\mathrm{x}},\bar{\mathrm{y}})| \leq \mathrm{P}_1(\mathrm{t})|\mathrm{x}-\bar{\mathrm{x}}| + \mathrm{P}_2(\mathrm{t})|\mathrm{y}-\bar{\mathrm{y}}|$, on the set $\mathrm{J}\times R^n \times R^n$, where $\mathrm{P}_1, \mathrm{P}_2\colon \mathrm{J} \to \mathrm{R}^{\mathrm{n}\times\mathrm{n}}$ are matrix functions with entries (piecewise) continuous on *I*. Then the initial value problem (7) has a unique solution $\mathrm{x}\colon I \to R^n$ and, for any continuous function $\mathrm{x}_0\colon \mathrm{J} \to R^n$ and any $\nu \in N$, the problem

$$\mathrm{x}_\nu'(\mathrm{t}) = \mathrm{f}\Big(\mathrm{t}, \mathrm{x}_\nu(\mathrm{t}), \chi_\mathrm{I}\big(\tau(\mathrm{t})\big)\mathrm{x}_{\nu-1}\big(\tau^0(\mathrm{t})\big) + \Big(1 - \mathrm{chi}_\mathrm{I}\big(\tau(\mathrm{t})\big)\Big)\mathrm{h}\big(\tau(\mathrm{t})\big)\Big), \qquad \mathrm{x}_\nu(0) = \mathrm{h}(0), \tag{8}$$

has a unique solution $\mathrm{x}_\nu\colon \mathrm{J} \to R^n$. Moreover,

$$\lim_{\nu\to+\infty} ||\,\mathrm{x}(\mathrm{t}) - \mathrm{x}_\nu(\mathrm{t})||_J = 0. \tag{9}$$

Proof

Consider first the mapping defined by the relation

$$p(\mathrm{u})(\mathrm{t}) = \mathrm{P}_1(\mathrm{t})\mathrm{u}(\mathrm{t}) + \chi_\mathrm{I}\big(\tau(\mathrm{t})\big)\mathrm{P}_2(\mathrm{t})\mathrm{u}\big(\tau^0(\mathrm{t})\big), \quad \mathrm{t} \in \mathrm{J}, \tag{10}$$

for any $\mathrm{u} \in \mathrm{C}(I; R^n)$, with P_1, P_2 and τ as above. Obviously, p is a linear mapping from $\mathrm{C}(I; R^n)$ to $\mathrm{L}(I; R^n)$. It is a Volterra operator (with respect to the point *0*) which is also strongly bounded (Kiguradze, 1997).

Then there exists a nonnegative Lebesgue integrable function η such that

$$\|\mathrm{l}(\mathrm{u})(\mathrm{t})\| \leq \eta(\mathrm{t})\|\mathrm{u}\|_{[0,\mathrm{t}]} \quad \text{for a.e. } \mathrm{t} \in \mathrm{J} \tag{11}$$

for any continuous $\mathrm{u} \in \mathrm{C}(I; R^n)$. Moreover, (11) implies that

$$\left\|l^k(\mathrm{u})(\mathrm{t})\right\| \leq \frac{1}{\mathrm{k}!}\left(\int_0^\mathrm{t} \eta(\mathrm{s})\mathrm{ds}\right)^\mathrm{k} \|u\|_{[0,\mathrm{t}]} \quad \text{for } \mathrm{t} \in J,\ \mathrm{k} \in N, \tag{12}$$

where $\|u\|_{[0,\mathrm{t}]} = \max\{\,\|\mathrm{u}(\mathrm{s})\|\colon \mathrm{s} \in [0;\mathrm{t}]$ and

$$\mathrm{l}^\mathrm{k}(\mathrm{u})(\mathrm{t}) = \int_0^t \mathrm{p}(p^{k-1}(\mathrm{u}))(\mathrm{s})\mathrm{ds}), \quad \mathrm{t} \in J, \tag{13}$$

for all $\mathrm{k} \in N$.

Let $\varphi \in \mathrm{C}(\mathrm{J}; R^n)$ and $\mathrm{c}_0 \in R^n$ be arbitrary. Define the function $\mathrm{f}_\varphi\colon \mathrm{J} \times R^n \to R^n$ by setting $\mathrm{f}_\varphi(\mathrm{t},\mathrm{x}) = \mathrm{f}\big(\mathrm{t},\mathrm{x},\varphi(\mathrm{t})\big), \quad \mathrm{t} \in J, \mathrm{x} \in R^n$.

Then f_φ satisfies the conduction $\left|\mathrm{f}_\varphi(\mathrm{t},\mathrm{x}) - \mathrm{f}_\varphi(\mathrm{t},\bar{\mathrm{x}})\right| \leq \mathrm{P}_1(\mathrm{t})|\mathrm{x}-\bar{\mathrm{x}}|$ on the set $\mathrm{J}\times R^n$. By analogy to (10) , define the operator p_φ by putting $p_\varphi(\mathrm{u})(\mathrm{t}) = \mathrm{P}_1(\mathrm{t})\mathrm{u}(\mathrm{t}), \quad \mathrm{t} \in J$, on $\mathrm{C}(J; R^n)$.

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Then l_φ is a strongly bounded Volterra operator with respect to 0 and, by analogy to, (12), we have

$$\left\|l_\varphi^k(u)(t)\right\| \le \frac{1}{k!}\left(\int_0^t \eta_\varphi(s)ds\right)^k \|u\|_{[0,t]} \quad t \in J, \tag{14}$$

for all $k = 0,1,...$, where η_φ is a function with properties similar to those of η and p_ϕ^k ($k \in N$) are defined similarly to (13). The conditions assumed ensure that both f and f_φ are Carathéodory functions.

Put

$$(Tx)(t) = h(0) + \int_0^t f\Big(s, x(s), \chi_I\big(\tau(s)\big)x\big(\tau^0(t)\big) + \Big(1-\chi_I\big(\tau(s)\big)\Big)h\big(\tau(s)\big)\Big)ds \tag{15}$$

$$\big(T_\varphi x\big)(t) = c_0 + \int_0^t f_\varphi\big(s, x(s)\big)ds, \quad t \in J, \tag{16}$$

for all x from $C(J, R^n)$. Then problem (7) is equivalent to

$$x = Tx \tag{17}$$

and, similarly, the initial value problem

$$x'(t) = f_\varphi\big(t, x(t)\big), \; x(0) = c_0, \quad t \in J, \tag{18}$$

is equivalent to the operator equation

$$x = T_\varphi x. \tag{19}$$

Take any x and $\bar{x}$ from $C(J; R^n)$. Then (13) and (15) imply that

$$|| \,(Tx)(t) \,-\, (T\bar{}\{x\})(t)\, || \,=\, \left\|\int_0^t [\, f(s, x(s)\chi_I\,(\tau(s))x(\tau^0(s)) + (1-\chi_I\,(\tau(s)))h(\tau(s))) - f(s,\bar{}\{x\}(s)\,\chi_I\,(\tau(s))\bar{}\{x\}(\tau^0(s)) + (1-\chi_I\,(\tau(s)))h(\tau(s)))]\,ds\right\| \le \left\|\int_0^t\big[P_1(s)|x(s)-\bar{x}(s)| + P_2(s)\,\chi_I\,(\tau(s)))\,x\big(\tau^0(s)\big) - \bar{x}\big(\tau^0(s)\big)ds\big]\right\| \,=\, \|l^1(|\,x \,-\, \bar{x}|)\,(t)\| \,\le\, \int_0^t \eta\,(s)ds\,\|x-\bar{x}\|_{[0;t]}, \quad t \in J,$$

whence

$$\left\|\big(T^{k_1}x\big)(t) - \big(T^{k_1}\bar{x}\big)(t)\right\| \le \left\|l^1\Big(\big|\big(T^{k_1-1}x\big) - \big(T^{k_1-1}\bar{x}\big)\big|\Big)(t)\right\| \le \cdots \le \left\|l^{k_1}(|x-\bar{x}|)(t)\right\| \le \frac{1}{k_1!}\left(\int_0^t \eta(s)ds\right)^{k_1}\|x-\bar{x}\|_{[0;t]}, \quad t \in J, \tag{20}$$

for any $k_1 \in N$.

Similarly, by (14),

$$\left\|\big(T_\varphi x\big)(t) - \big(T_\varphi \bar{x}\big)(t)\right\| \le \left\|p_\varphi^1(|x-\bar{x}|)(t)\right\| \le \int_0^t \eta_\varphi(s)ds\,\|x-\bar{x}\|_{[0;t]}, \quad t \in J, \tag{21}$$

whence

$$\left\|\Big(T_\varphi^{k_2}x\Big)(t) - \Big(T_\varphi^{k_2}\bar{x}\Big)(t)\right\| \le \left\|l_\varphi^{k_2}(|x-\bar{x}|)(t)\right\| \le \frac{1}{k_2!}\left(\int_0^t \eta_\varphi(s)ds\right)^{k_2} \tag{22}$$

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$$\|x-\bar{x}\|_J, \quad t \in J,$$

for any $k_2 \in N$. Since

$$\lim_{k\to+\infty} \frac{1}{k!}\left(\int_0^t \eta(s)ds\right)^k = \lim_{k\to+\infty} \frac{1}{k!}\left(\int_0^t \eta_\varphi(s)ds\right)^k = 0, \tag{23}$$

it follows from (20), (21) that there exist k_1 and k_2 from N for which T^{k_1} and $T_\varphi^{k_2}$ are contraction mappings. Using in (17),(19) the theorem on the fixed point of an operator T such that T^m is a contraction mapping for some m, we show that the initial value problems (7) and (18) are uniquely solvable.

Put in (18) $\varphi(t) = \chi_I(\tau(s))x_{\nu-1}(\tau^0(t)) + \left(1-\chi_I(\tau(s))\right)h(\tau(s))$, $c_0 = h(0)$, with $x_0 \in C(I;R^n)$ where, for any $\nu \in N$, x_ν stands for the solution of (18). Then $\{x_\nu\}_{\nu=1}^{\infty}$ is the sequence of solutions of problems (8).

It remains to prove (9), where x is the unique solution of problem (7). Let $\{\overline{x_\nu}\}_{\nu=1}^{\infty}$ be the sequence of successive approximations corresponding to the equivalent equation (17), where $\overline{x_0} \in C(I;R^n)$ is arbitrary. Then

$$\begin{aligned} &\|x(t)-x_\nu(t)\| \le \|x(t)-\overline{x_\nu}(t)\| + \|\overline{x_\nu}(t)-x_\nu(t)\| \\ &\le \int_0^t \eta(s)ds\,\|x-\overline{x_\nu}\|_{[0;t]} + \|(T\overline{x_{\nu-1}})(t)-(Tx_{\nu-1})(t)\| \\ &\le \int_0^t \eta(s)ds\,\|x-\overline{x_\nu}\|_J + \int_0^t \eta(s)ds\,\|x_{\nu-1}-\overline{x_{\nu-1}}\|_{[0;t]} \\ &\le \int_0^t \eta(s)ds\,\|x-\overline{x_\nu}\|_J + \frac{1}{\nu!}\left(\int_0^t \eta(s)ds\right)^k \|x_0-\overline{x_0}\|_J, t \in J \end{aligned} \tag{24}$$

Since $\lim_{\nu\to\infty}\|x-\overline{x_\nu}\|_I = 0$, it follows from (23) and (24) that (9) holds.

Note that the iteration process described above is stable in a certain sense: if, along with problems (8), $\nu = 1,2,...$, we consider the sequence of problems

$$\overline{x_\nu'}(t) = f\left(t, \overline{x_\nu}(t), \chi_I(\tau(t))\overline{x_{\nu-1}}(\tau^0(t)) + \left(1-\chi_I(\tau(t))\right)h(\tau(t))\right) + g_\nu(t), t \in J,$$

$$\overline{x_\nu}(0) = h(0) + c_\nu,$$

For $\nu = 1,2,$, where $\lim_{\nu\to\infty}\left(c_\nu + \int_0^{t_r} g_\nu(s)ds\right) = 0$, then $\lim_{\nu\to\infty}|x_\nu - \overline{x_\nu}|_I = 0$ i.e., under the conditions assumed, the sequence $\overline{x_\nu}$converges to the solution x of problem (7).

5. Numerical experiment

Here we present examples illustrating the above theory and demonstrating the behavior of the model under various conditions in the context of the stock exchange environment. Numerical solutions and graphical presentation of the solutions of the model were carried out in Maple, which provides all necessary tools for simulation of real problems.

Let us denote the rate of signal change as d_3, which is related to the volatility index VIX (or, in layman's terms, the fear index).

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In order to calculate the solution of our model, we assume that we know the "historical development" before $t = 0$ (the time unit shall be one day for us), which can be, for the sake of simplicity, simulated using constant functions $\mathrm{I_h(t)} = 230000$ (the number of individuals); $\mathrm{I_h^*(t)} = 20$; (the number of individuals); $\mathrm{S_h(t)} = 40600$ (monetary unit). We will further assume that prospective investors need time $\tau = 1{,}25\sin(\mathrm{t})$ (day) to evaluate the information. The graphical simulation will be presented on daily interval, i.e. an interval longer than one year.

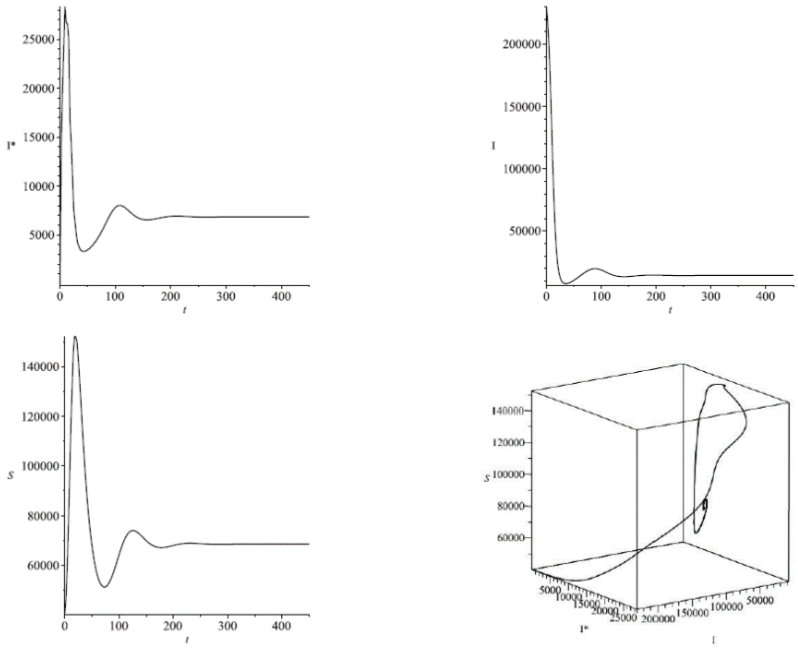

Figure 1. Solution of numerical experiment I

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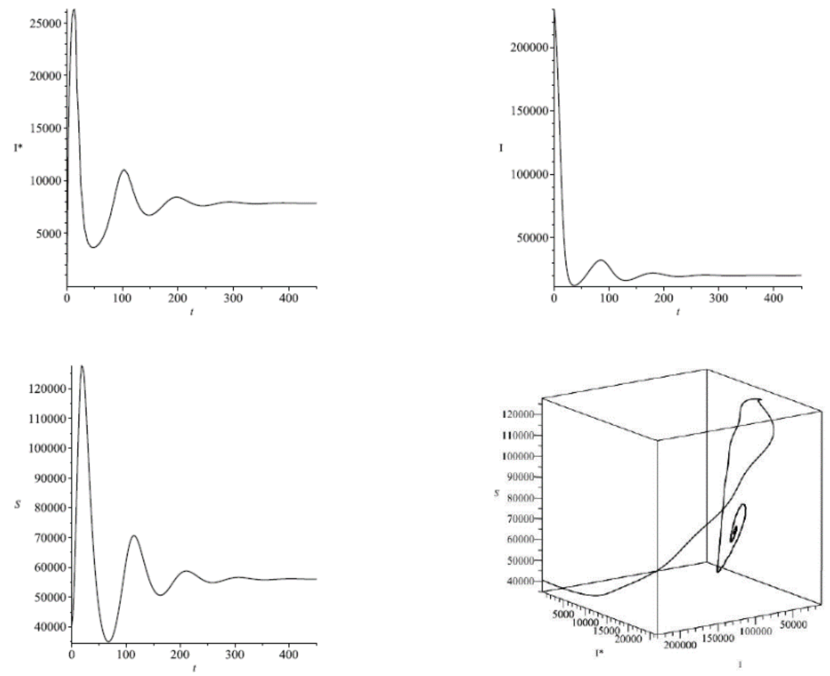

Figure 1. Solution of numerical experiment II

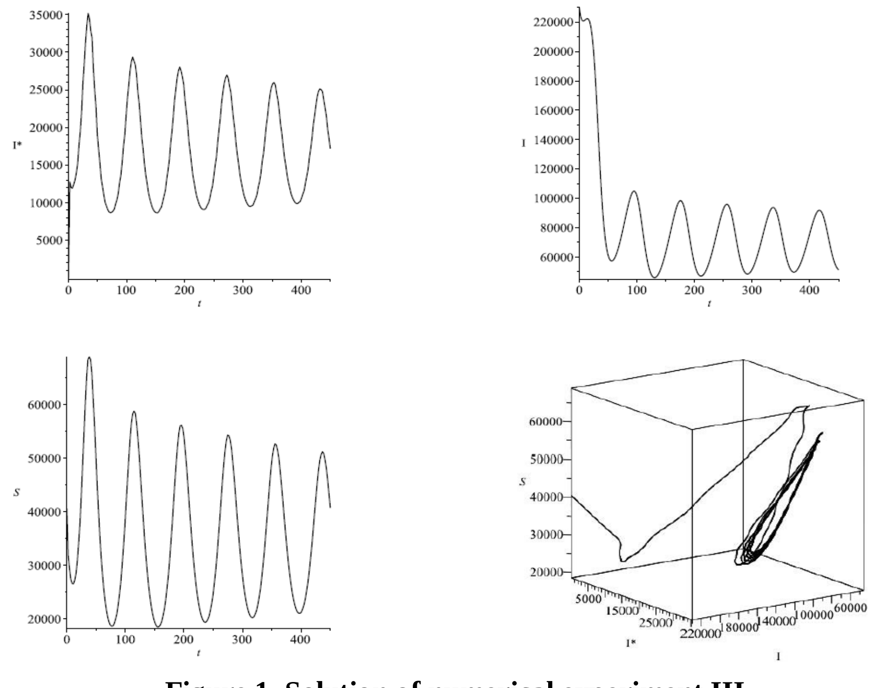

Figure 1. Solution of numerical experiment III

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I) The following setup of parameters shall be considered in the initial situation: $p = 1000$ (the number of individuals); $q = 0{,}01$; $\mathrm{I}_{\max} = 20000000$ (the number of pieces); $\beta = 0{,}0000016$; $\mathrm{k} = 0{,}5$; $\mathrm{d}_3 = 0{,}05$; $\xi = 0{,}05$. The graph of the solution can be seen on Figure 1. Low d_3 corresponds to market conditions of long-term low volatility. In the environment of "stable" stock markets, many small investors buy shares after they have been launched on the primary market. Only a small number of these investors buy with a short-term horizon for speculation purposes (i.e. they will sell these shares in a short time). Such behavior of retail investors helps to stabilize the price of the newly traded stock.

II) We now assume that the rate of signal change has increased to $\mathrm{d}_3 = 0{,}07$. The system (2) is asymptotically stable, as shown in Figure 2. Increased d_3 corresponds to market conditions of long-term increased volatility. In the environment of "nervous" stock markets, only some retail investors buy shares after they have been launched on the primary market (compared to the previous situation, or the previous number of retail investors). Some of these investors buy with a short-term horizon for speculation purposes (i.e. they will sell these shares in a short time). Such behavior of retail investors postpones stabilization of the price of this newly traded stock, compared to the previous case.

III) On the other hand, in case of yet another increase in the rate of signal change to $\mathrm{d}_3 = 0{,}24$, we obtain not asymptotically stable, which is also apparent in Figure 3. High d_3 corresponds to market conditions of high to extreme volatility. In this environment of "turbulent" stock markets, only speculative retail investors buy shares after they have been launched on the primary market. Such behavior of retail investors prevents the price of the newly traded stock from stabilizing.

6. Discussion

The fact that at first glance simple but nonlinear systems may show very complicated and seemingly random behavior has been known for more than a century, but effective tools for solving differential equations typically used to describe them were lacking. In the preceding sections, we presented a way to solve such a problem using the modern theory of functional differential equations.

In order to continue working with our model, we needed to explore its behavior. That is why in the previous paragraph we confirmed, by simulation, that the solution presented by our model is positive.

Furthermore, based on the graphical solution presented, we may claim that numeric examples I and II show a situation after a company's IPO, when more and more investors (especially private ones) buy the company's shares in the first trading days. These increases are also speculative, so after a short period of time (days or weeks), some private investors begin to sell the shares. Figures 1. and 2. show that the number of potential investors plummets over the first few weeks of trading, while the number of investors increases in the short term. At the same

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time, while interest in the share is decreasing, we can observe an increase in the intensity of signal, which, however, has a significantly weaker response compared to that recorded in the first days.

Approximately at the end of the first year of trading, the number of investors stabilizes, which is associated with the first annual reporting of the company's economic performance on the stock exchange, and the comparison of the predicted economic results (stated in the Road Show process) with the real ones. At the same time, the intensity of signal stabilizes at a constant level.

Example III demonstrates what may happen if a company has not defined its strategy clearly enough, the strategy is not even properly communicated to the investors, managers of the share issue spend increasingly more on advertising, and, not having attracted much interest in its shares, the company markets itself inappropriately; therefore, the numbers of both the investors and prospective investors fluctuate and the company is gradually becoming untrustworthy.

7. Conclusion

The Stock Exchange is an important dynamic part of the capital market and a crucial segment for the development of joint-stock companies. Given global development, it may be assumed that with further development of the capital market there will be a significant shift in the use of initial public offerings as an interesting financial instrument that can bring in financial resources to businesses and be an attractive investment for investors. Putting new shares into circulation brings profit to the entire economy as well, because strong companies are the basis for thriving development. The equity earned through the stock exchange goes to companies which use it for large investments. This makes companies more competitive, existing jobs are better-paid, and new jobs are created.

It is not only for the above-mentioned reasons that it might be considered important to examine behavior in the stock exchange environment from different perspectives (benefits for issuers, investors, as well as to the national economy). In order to describe the behavior of individual actors during an IPO and, in particular, how the shares market price is determined, we used dynamic models and analogy with models of epidemic (viral) processes.

Dynamic models are a powerful tool especially for analyzing dynamic and evolving systems, which the capital market undoubtedly is.

The main contribution of our paper is a new perspective on solving the problem of dissemination of information and the application of a model from the field of biology in the examination of economic systems. An analogy with modelling of epidemic (viral) processes was applied to dissemination of information about a new share issue after the issue has been introduced to prospective investors. The outcome is a proposal for a new method of constructing a solution of a dynamic model using the theoretical apparatus of virus spread (similar to information about a new share issue) describing in particular the behavior of both investors and the issuer.

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Based on the results obtained, we can observe that the model and results of modelling show consequences of the behavior of a system as a whole in different situations, both from the point of view of the issuer and the investor, and with the help of its dynamics, one can easily understand the mechanism of information dissemination and price formation in the IPO process stock exchanges.

Via the presented mathematical models and proposed design of its numerical solution, this study may provide a tool for solving problems from areas as varied as economics, finance, information management, crisis management, stock trading, or strategic management. Experimental results obtained from the model confirm the efficiency and reliability of the proposed method. Therefore, we hope that the results obtained in this paper will help, in the future, investigate the behavior of other nonlinear dynamic models.

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